Section 5.4: Properties of rational functions

## Horizontal Asymptotes

To find the horizontal asymptote, we compare the degree of the numerator with the degree of the denominator.

$$
f(x)=\frac{a x^{n}+\ldots}{b x^{m}+\ldots}
$$

If $\mathrm{n}<\mathrm{m}$ then horizontal asymptote is the x -axis $(y=0)$.
If $\mathrm{n}=\mathrm{m}$ then the horizontal asymptote is $y=\frac{a}{b}$.
If $n>m$ then there is no horizontal asymptote. (There is an oblique asymptote.)

1) $f(x)=\frac{2 x-6}{x+2}$

Use the graph of $f(x)$ the function to find the following, then confirm your answer using Algebra.
a) find the domain, express your answer using interval notation
b) find the equation of the vertical asymptote(s)
c) find the equation of the horizontal asymptote
d) find the $x$-intercept
e) find the $y$-intercept

2) $f(x)=\frac{4 x-12}{x+2}$

Use the graph of $f(x)$ the function to find the following, then confirm your answer using Algebra.
a) find the domain, express your answer using interval notation
b) find the equation of the vertical asymptote(s)
c) find the equation of the horizontal asymptote
d) find the $x$ - intercept
e) find the $y$-intercept

3) $f(x)=\frac{4 x-12}{2 x+12}$

Use the graph of $f(x)$ the function to find the following, then confirm your answer using Algebra.
a) find the domain, express your answer using interval notation
b) find the equation of the vertical asymptote(s)
c) find the equation of the horizontal asymptote
d) find the $x$ - intercept
e) find the $y$-intercept

4) $f(x)=\frac{8 x-16}{2 x+16}$

Use the graph of $f(x)$ the function to find the following, then confirm your answer using Algebra.
a) find the domain, express your answer using interval notation
b) find the equation of the vertical asymptote(s)
c) find the equation of the horizontal asymptote
d) find the $x$-intercept
e) find the $y$-intercept

5) $f(x)=\frac{x-14}{x^{2}+6 x-7}$

Use the graph of $f(x)$ the function to find the following, then confirm your answer using Algebra.
a) find the domain, express your answer using interval notation
b) find the equation of the vertical asymptote(s)
c) find the equation of the horizontal asymptote
d) find the $x$ - intercept
e) find the $y$-intercept

6) $f(x)=\frac{x-10}{x^{2}+4 x-5}$

Use the graph of $f(x)$ the function to find the following, then confirm your answer using Algebra.
a) find the domain, express your answer using interval notation
b) find the equation of the vertical asymptote(s)
c) find the equation of the horizontal asymptote
d) find the $x$ - intercept
e) find the $y$-intercept

7) $f(x)=\frac{x-4}{x^{2}-11 x-12}$

Use the graph of $f(x)$ the function to find the following, then confirm your answer using Algebra.
a) find the domain, express your answer using interval notation
b) find the equation of the vertical asymptote(s)
c) find the equation of the horizontal asymptote
d) find the $x$ - intercept
e) find the $y$-intercept

8) $f(x)=\frac{x-9}{x^{2}-3 x-18}$

Use the graph of $f(x)$ the function to find the following, then confirm your answer using Algebra.
a) find the domain, express your answer using interval notation
b) find the equation of the vertical asymptote(s)
c) find the equation of the horizontal asymptote
d) find the $x$ - intercept
e) find the $y$-intercept

9) $f(x)=\frac{x^{2}+5 x-6}{x+3}$

Use the graph of $f(x)$ the function to find the following, then confirm your answer using Algebra.
a) find the domain, express your answer using interval notation
b) find the equation of the vertical asymptote(s)
c) find the equation of the SLANT asymptote
d) find the $x$ - intercept
e) find the $y$-intercept

10) $f(x)=\frac{x^{2}+6 x-7}{x+1}$

Use the graph of $f(x)$ the function to find the following, then confirm your answer using Algebra.
a) find the domain, express your answer using interval notation
b) find the equation of the vertical asymptote(s)
c) find the equation of the SLANT asymptote
d) find the $x$ - intercept
e) find the $y$-intercept

11) $f(x)=\frac{x^{2}-6 x-16}{x-2}$

Use the graph of $f(x)$ the function to find the following, then confirm your answer using Algebra.
a) find the domain, express your answer using interval notation
b) find the equation of the vertical asymptote(s)
c) find the equation of the SLANT asymptote
d) find the $x$ - intercept
e) find the $y$-intercept

12) $f(x)=\frac{x^{2}-3 x-4}{x-1}$

Use the graph of $f(x)$ the function to find the following, then confirm your answer using Algebra.
a) find the domain, express your answer using interval notation
b) find the equation of the vertical asymptote(s)
c) find the equation of the SLANT asymptote
d) find the $x$ - intercept
e) find the $y$-intercept

23) $f(x)=\frac{x+3}{x^{2}-9}$

Find the following:
a) equation of the vertical asymptote
b) coordinates of the "hole" in the graph of $f(x)$

14) $f(x)=\frac{x+2}{x^{2}-4}$

Find the following:
a) equation of the vertical asymptote
b) coordinates of the "hole" in the graph of $f(x)$

15) $f(x)=\frac{x+2}{x^{2}-6 x-16}$

Find the following:
a) equation of the vertical asymptote
b) coordinates of the "hole" in the graph of $f(x)$

16) $f(x)=\frac{x+3}{x^{2}-x-12}$

Find the following:
a) equation of the vertical asymptote
b) coordinates of the "hole" in the graph of $f(x)$

\#17-28:
For each problem find the following:
a) the domain of $f(x)$ written in interval notation
b) the equation of the vertical asymptote (write none if there is no vertical asymptote)
c) the equation of the horizontal asymptote (write none if there is no horizontal asymptote)
d) the equation of the slant asymptote (write none if there is no slant asymptote)
e) write the coordinates of any "hole" (write none if there is no hole)
f) $x$ - intercept(s) if any
g) $y$-intercept(s) if any
h) Sketch a graph of the function
17) $f(x)=\frac{4 x+12}{x-3}$
19) $f(x)=\frac{6 x-24}{2 x+12}$
21) $f(x)=\frac{x-14}{x^{2}+6 x-7}$
23) $f(x)=\frac{x-5}{x^{2}-25}$
25) $f(x)=\frac{x^{2}+6 x-16}{x-4}$
27) $f(x)=\frac{x^{2}+3 x-18}{x-1}$
18) $f(x)=\frac{6 x-18}{x+2}$
20) $f(x)=\frac{8 x-24}{2 x+12}$
22) $f(x)=\frac{x-16}{x^{2}+7 x-8}$
24) $f(x)=\frac{x-6}{x^{2}-36}$
26) $f(x)=\frac{x^{2}+5 x-6}{x+2}$
28) $f(x)=\frac{x^{2}+2 x-24}{x-1}$

